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Incomplete State Feedback for Time Delay Systems: Observer Applications in Multidelay Compensation

This paper demonstrates how a recently developed observer for time delay systems may be used to estimate needed state variables for implementation of multivariable time delay compensation. The general results are illustrated by an example of a multireactor plant in which only one reactor concentration can be measured. The observer worked well in simulation for both multivarible PID control and multidelay compensated PID control and allowed both schemes to function with estimated state variables in the feedback loop.

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SCOPE

The problem of incomplete measurement for feedback control is a serious difficulty in the design of process control systems. This paper discusses how limited primary and secondary measurements may be integrated into an estimator for systems having time delays. The estimator is demonstrated for multiloop PID control and for a control system employing multidelay compensation.

CONCLUSIONS AND SIGNIFICANCE

Techniques for estimation of unmeasured states in systems with time delays are presented and illustrated by example. The simulation results indicate that the estimator should perform well when put in tandem with a multidelay compensator or other multivariable control scheme.

INTRODUCTION

In many industrial processes, transportation lags in pipes, long recycle loops, and sampling and analysis delays are frequently responsible for introducing time delays into the overall control system. These time delays very often inhibit good control system performance.

As a result control algorithms which specifically take the presence of time delays into consideration have been proposed. For systems having multiple time delays, the multidelay compensator has been developed (Ogunnaike and Ray, 1979) and has been shown to work well on an experimental system (Ogunnaike et al., 1981)

For the implementation of this time delay compensator it is tacitly assumed that all the required measurements can be made available to the controller. For systems described by input/output relations (as in the frequency domain with transfer function matrices), this assumption usually holds. In the event some of the outputs are not measured on-line, there must be other auxiliary outputs from which the unmeasured ones can be inferred (e.g., product compositions from tray temperature measurements in distillation). This requires the use of some simple correlating

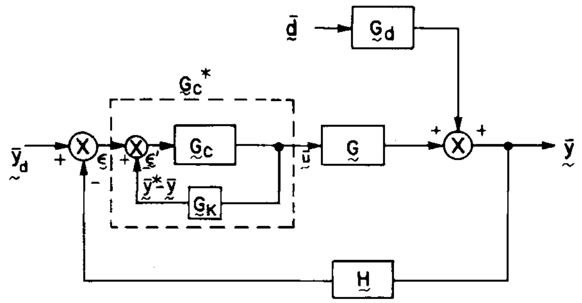


Figure 1. Block diagram of feedback control of multivariable system with time delay compensation.

relations which will be discussed later. However, for systems described in state-space, such as differential equations derived from the systematic application of physical laws (e.g., material and energy balances and other related considerations), some of the states which are represented in the model might not be available for measurement. We shall consider this type of model first.

It has been the practice, when the total state vector cannot be measured, to call upon on-line state estimation techniques which will provide acceptable estimates of the missing state variables (cf., Ray, 1980). If the state estimation is to be done in the face of measurement error, the device which is normally utilized is the Kalman filter (Kalman and Bucy, 1961), while in a deterministic case, the Luenberger observer is used (cf., Luenberger 1964, 1966, 1971). Extending filtering theory to encompass systems with time delays has been done (Kwakernaak, 1967), and extensions of observer theory to this case have also been carried out (Hewer and Nazaroff, 1973; Gressang and Lamont, 1975; Bhat and Koivo, 1976; Ogunnaike, 1981a).

When the multidelay compensator shown in Figure 1 is to be implemented for a system with incomplete measurement, additional considerations are called for. We recall that the compensator is required to produce $(y^* - y)$, where y is a simulation of the actual system output (obtained from the system model) and y^* is the output for the corresponding undelayed version of the system model. When information about the state vector is incomplete, a state estimator is used because the initial conditions of the states are not completely known. If these initial conditions were known the system model could be used, entirely by itself, to produce estimates of the missing information. However, the delay compensator only produces the required signals $(y^* - y)$ from the system model. Clearly then, when some states are unavailable for measurement, the delay compensator can no longer function as originally intended.

The purpose of this paper is to present results on how to implement the time delay compensator for incomplete state feedback problems. Thus both problems (i) providing estimates for unmeasured outputs (so that feedback control can be done) and (ii) the implementation of the time delay compensator block (to produce $y^* - y$) will be discussed. An observer is introduced into the closed loop to provide estimates of the missing outputs *both* to the controller, and to a device called the compensator simulator which utilizes the observer information to produce estimates of $(y^* - y)$. These estimates are shown to converge on the actual, but unknown variables. The observer theory of Ogunnaike (1981a) will be recalled and utilized, although the development is in such a way that any observer at all which yields asymptotically stable estimates can equally well be used along with the compensator simulator. A comprehensive example incomplete state feedback problem, such as is commonly encountered in engineering practice, is addressed to illustrate these results.

OBSERVER APPLICATIONS IN MULTIDELAY COMPENSATION

For linear systems having time delays, it is possible to prove that a separation theorem exists (Koivo and Koivo, 1978). Therefore, in designing feedback control systems for incomplete state information it is possible to design the controller assuming that all the state variables are available; then an observer may be incorporated for the purpose of producing asymptotic estimates of the missing states. We will briefly recapitulate observer techniques for time delay systems, the design of delay compensators when *all* measurements are available, and then we will introduce the compensator simulator which makes it possible to implement the delay compensator when some states are missing.

Time Delay System Observers

Owing to the fact that the pioneering works of Hewer and Nazaroff (1973) and Gressang (1974) in extending Luenberger theory to time delay systems yielded observers whose stability characteristics were quite tedious to check, other approaches have been proposed since then.

The approach of Gressang and Lamont (1975) and later Bhat and Koivo (1976) which made use of spectral decomposition techniques yielded observers with rigorously established stability properties. However, in the theoretical developments it was necessary to assume a single delay to somewhat simplify the analysis, but these results could be extended to the multiple delay case, even if with considerable effort.

The approach of Ogunnaike (1981a) involved the introduction of operator matrices (so that multiple delays can be considered with ease) and a judicious resolution of the state vector into two component vectors so that a direct appeal could be made to Luenberger theory. This approach yielded an observer whose eigenvalues can be specified by the designer (thus guaranteeing stability) as well as being quite simple to design and implement.

Consider the time delay system represented as

$$\dot{x} = \sum_i A_i x(t - \rho_i) + \sum_j B_j u(t - \beta_j) \tag{1}$$

if we define B as the backshift operator according to

$$B^{\alpha} x(t) = x(t - \alpha) \quad \text{(cf., Box and Jenkins, 1976)} \tag{2}$$

then in the notation of Ogunnaike (1981a), Eq. 1 becomes, in *Operator-Matrix notation*.

$$\dot{x} = \mathcal{A} x + \mathcal{B} u \tag{3}$$

where

$$\mathcal{A} = \sum_i A_i B^{\rho_i}, \quad \mathcal{B} = \sum_j B_j B^{\beta_j} \tag{4}$$

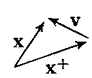
We recall the proposed observer theory as involving the partitioning of x into variables x_o which are "observed" (either through direct measurement or determination from $y = Cx$) and variables x_u which are not observed. Hence

$$x = \begin{bmatrix} x_o \\ \hline x_u \end{bmatrix} \begin{matrix} \text{observed subspace} \\ \text{unobserved subspace} \end{matrix}$$

which results in a corresponding partitioning of $\mathcal{A}$ and $\mathcal{B}$ as

$$\mathcal{B} = \left[\begin{array}{c|c} \mathcal{A}_{oo} & \mathcal{A}_{ou} \\ \hline \mathcal{A}_{uo} & \mathcal{A}_{uu} \end{array}\right] \quad \mathcal{B} = \begin{bmatrix} \mathcal{B}_o \\ \mathcal{B}_u \end{bmatrix} \tag{5}$$

When the vector x is resolved as shown below



where x^+ is the state vector of the system

$$\dot{x}^+ = \mathcal{A}^+ x^+ + \mathcal{B} u$$

and v is from the free "complementary system"

$$\dot{v} = \Omega v$$

it was shown that if

$$\mathcal{A}^+ = \left[\begin{array}{c|c} \mathcal{A}_{oo} & \mathcal{A}_{ou} \\ \hline \mathcal{A}_{uo} & K \end{array}\right] \quad \text{and } \Omega = \left[\begin{array}{c|c} \mathcal{A}_{oo} & \mathcal{A}_{ou} \\ \hline \mathcal{A}_{uo} & \Phi \end{array}\right] \tag{6}$$

$$\text{with } \Phi = (\mathcal{A}_{uu} R - K)(R - I)^{-1} \tag{7}$$

then the observer equations for estimating x_u are

$$\dot{z} = Kz + \mathcal{A}_{uo}(y - \eta) + \mathcal{B}_u u$$
$$x_u = Rz \tag{8}$$

for constant matrices R and K.
where $y = Cx$ is the available system output
$\eta = Cv$ is the output from the complementary system (Ogunnaike, 1981a)

The observer open loop stability was shown to be dependent on K, a matrix whose elements are chosen by the designer. In addition, for a feedback law

$$u = G_c \hat{x}$$

where $\hat{x}$ are the observer estimates, it was also shown (Ogunnaike, 1981a) that the closed loop stability is determined from the following union of spectra:

$$\sigma(\mathcal{A} + \mathcal{B}G_c) U \sigma(K).$$

And since K is designer-specified, the observer will be stable in the closed loop if the chosen controller G_c *does not* make the overall plant-controller system unstable. However it is known that choosing acceptable G_c matrices for multivariable time delay systems can be difficult because improving sluggish responses through the use of high controller gains frequently results in system stability being jeopardized. However, when the multidelay compensator (cf., Ogunnaike and Ray, 1979) is used in conjunction with the controllers, higher controller gains can be used without sacrificing system stability—facilitating the choice of acceptable G_c matrices.

Multidelay Compensation

Suppose that the general time delay system of Eq. 1 has outputs (or measurements) given by

$$y = Cx. \tag{9}$$

When C is so structured that all the states required for control are available through measurement (either C is the identity matrix or it is invertible), the control problem merely involves designing the time delay compensator block which, according to Figure 1 produces $(y^* - y)$.

It is clear from its definition that for the system in Eq. 1 the corresponding y^* is produced from

$$\dot{x}^* = A^* x^*(t) + B^* u(t) \tag{10}$$

with

$$y^* = Cx^*$$

where $A^* = \Sigma_i A_i$, $B^* = \Sigma_j B_j$ are obtained by setting $\rho_i = \beta_j = 0$, or equivalently

$$A^* = \mathcal{A}|_{B=0} \quad B^* = \mathcal{B}|_{B=0} \tag{11}$$

When some of the required states are unavailable through measurement the results in Ogunnaike (1981a) (or any other adequate observer) can be called upon in designing an observer to produce asymptotic estimates of these missing states. *We note however that the implementation of the compensator block requires y^* along with y.*

An observer is needed in order to obtain the full y vector whenever initial conditions for the entire state vector x are not known; otherwise estimates of y could be produced solely from the system model. The same inhibition therefore holds for y^*. Whenever an observer has to be used to estimate the missing y's, the corresponding y^*'s must also be estimated since they are no longer reproducible solely from the system model. Rather than design 2 observers for this purpose, a solution to the problem is presented in the next section which involves the introduction of a dynamical system which synthesizes $(y^* - y)$ as a single vector w from the observer outputs x and the process inputs u. The problem of individually producing y and y^* is thus circumvented.

Compensator Simulator

It is easy to show that the dynamical system

$$\dot{w} = \Lambda w + \mathcal{D}x + \mathcal{P}u \tag{12}$$

simulates the vector

$$w = y^* - y \tag{13}$$

when the matrices are as defined below:

$$\Lambda = CA^*C^{-1}$$
$$\mathcal{D} = C(A^* - \mathcal{A}) \tag{14}$$
$$\mathcal{P} = C(B^* - \mathcal{B})$$

with an initial condition $w(0) = 0$. To show this one may differentiate Eq. 13 to yield

$$\dot{w} = C(\dot{x}^* - \dot{x})$$

or from Eqs. 3 and 10

$$\dot{w} = CA^*x^* + CB^*u - C\mathcal{A}x - C\mathcal{B}u \tag{15}$$

from Eq. 13

$$x^* = C^{-1}w + x \tag{16}$$

Equation 15 becomes upon rearrangement

$$\dot{w} = CA^*C^{-1}w + C(A^* - \mathcal{A})x + C(B^* - \mathcal{B})u \tag{17}$$

as is required. It should be intuitively obvious from the definition of y^* that it has identical initial conditions with y [hence $w(0) = 0$] and the dynamical system operating according to Eq. 12 simulates the compensator block of Figure 1 making it possible for the scheme to be implemented when the observer estimates y have to be used in place of unavailable measurements.

Note that in simulating Eq. 12, observer estimates $\hat{x}$ would have to be used in place of x. This implies that the output w will contain some errors which would be transmitted to the controller. We now show that these errors are guaranteed to vanish as $t \to \infty$.

The compensator simulator of Eq. 12 yields asymptotically stable estimates of $w = y^* - y$ in the sense that errors e_w resulting from using observer estimates $\hat{x}$ instead of x vanish as $t \to \infty$, given an open-loop system which is stable or stabilizable by an inner loop feedback controller. To show this we recall that estimates of $(y^* - y)$ are produced according to

$$\dot{\hat{w}} = \Lambda \hat{w} + \mathcal{D}\hat{x} + \mathcal{P}u \tag{18}$$

Thus subtracting Eq. 18 from Eq. 12 yields

$$\dot{e}_w = \Lambda e_w + \mathcal{D}e_x \tag{19}$$

Observe that in the above equation, the 'input' e_x is bounded (because the observer guarantees $\hat{x} \to x$). Furthermore, by definition, Λ and A^* have the same eigenvalues. (See Eq. 14 and Gantmacher, 1959.) If the system is open-loop stable or stabilizable, the eigenvalues of A^* (& hence of Λ) have negative real parts.

Thus, boundedness of e_x and the eigenvalues of Λ being negative combine to dictate that Eq. 19 has a steady state solution given by

$$e_w = -\Lambda^{-1}\mathcal{D}e_x \tag{20}$$

But at steady state

$$e_x = 0, \text{ by the design of the observer}$$

and hence Eq. 20 becomes

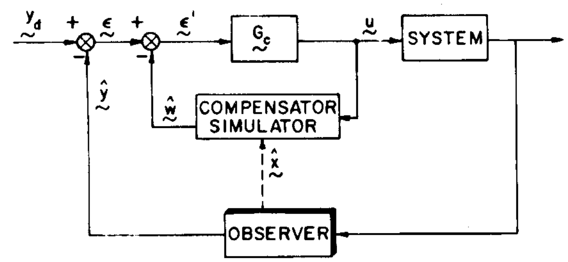


Figure 2. The multidelay compensator for incomplete state feedback.

$$e_w = 0 \text{ as required.}$$

In other words, since the observer with proper design guarantees that $\hat{x} \rightarrow x$ quickly the error introduced into the system performance through $\hat{w}$ also dies out quickly. (The example problem to follow demonstrates this point.)

The overall implementation of the scheme is summarized in the block diagram shown in Figure 2.

OBSERVER APPLICATIONS IN INFERENTIAL CONTROL

When systems are described by input/output transfer function matrices the question of the availability of measurements for control purposes hardly arises. This is because the existence of this class of models is directly dependent on the fact that the required outputs are measured. However, there are instances in which certain outputs are not available on-line. Under these circumstances off-line measurements can be made for the purpose of modeling, but for on-line computer control suitable secondary measurements (which are available on-line) must be made such that the unavailable outputs can be inferred from these. An example chemical engineering system in which these conditions can exist is the distillation column. Product compositions are not as easily available on-line as tray temperatures in which case on-line temperature measurements could be used for inferential control of compositions.

Transfer function matrix relations.

Let z be the system outputs desired to be controlled but which are not as easily available on-line as v, an m-vector of auxiliary (auxiliary in the sense that they are not the outputs desired to be controlled.) outputs. If y is the combined output vector $[z/v]$ of dimension n, and a transfer function matrix G exists between y and the system inputs u

$$\text{i.e.} \quad y = Gu \tag{21}$$

then it is possible to obtain a relationship to be used in inferring z from v *whenever there are at least as many auxiliary variables as there are inputs.*

Observe that Eq. 21 can be rewritten as

$$y = \begin{bmatrix} z \\ -- \\ v \end{bmatrix} = \begin{bmatrix} G_1(s) \\ ---- \\ G_2(s) \end{bmatrix} u \tag{22}$$

or

$$z = G_1(s)u \tag{23}$$

$$v = G_2(s)u \tag{24}$$

And if u is also an m-vector, then provided that G_2 is nonsingular, we have, from Eqs. 23 and 24

$$z = G_1(s)G_2^{-1}(s)v \tag{25}$$

which could be used to infer z given v from on-line measurements.

Remarks

1. Since y is n-dimensional and u is m-dimensional (as is v) then G_1 is an $(n-m) \times m$ matrix; and it is of no consequence if $(n-m)$ (the dimension of the required z vector) is less or greater than m. Note that the expression 25 is independent of the relative sizes of the z and v vectors; it only requires a nonsingular G_2.
2. However note that when $m > 3$, the inversion of $G_2(s)$ could become quite tedious.
3. The fact that $G_2(s)$ may contain time delays raises the question of realizability of Eq. 25 because of the presence of $G_2^{-1}(s)$.
4. All of the Laplace Domain treatment above requires that the initial conditions on *both* v and z be known. For v which is normally measured on-line this is usually not a problem, but for z it may be a problem in practice.

Observer Applications

The above mentioned complications that occur with the use of transfer function matrix relations for inference can be circumvented by utilizing an observer, as outlined below.

Given the complete system model

$$y = Gu$$

it has been shown (cf., Ogunnaike, 1981b) that such systems have corresponding state space realizations given by

$$\dot{x} = Ax + \sum_j B_j u(t - \beta_j)$$

$$y = Cx, \quad \text{i.e.:} \begin{bmatrix} z \\ - \\ v \end{bmatrix} = \begin{bmatrix} C_1 \\ - \\ C_2 \end{bmatrix} x \tag{26}$$

Because the delays appear only in the controls it is easy to show (cf. Ogunnaike, 1981b) that observer design for the system in Eq. 26 is the same as for a system with no time delays.

The observability question in this case simply reduces to investigating the rank of the G_2 matrix. If G_2 is nonsingular, the system is observable (cf., Ray, 1980 and Eq. 26). A full order observer (cf., Luenberger, 1966, 1971; Ray, 1980) can then be designed for the system in Eq. 27 from which inference about z can be made. Obviously a filter could also be used for estimation if statistical noise were a problem.

EXAMPLE PROBLEM

Network of Three Cascaded Reactors

A chemical company produces and markets the product P from reactant A. The isomerization reaction $A \rightarrow P$ is first order and essentially irreversible as well as isothermal. From reactor design considerations, a cascade of reactors is favored over a single one. Since the product P has three major market outlets requiring P at different concentrations, three stages of CSTR's are used. (The product from each reactor can thus be directly marketed; circumventing the need for an additional blending stage.) A diagram of the reactor system is shown in Figure 3. To conserve the expensive reactant A, product from reactors 2 and 3 are recycled as shown. (These recycle streams also make it easier to adjust when major changes in operating conditions are called for by market requirements.)

The concentrations C_1, C_2, C_3 of the product from each reactor are the vital variables to be controlled. This is done by manipulating the feed composition to the reactor network C_{1f}, C_{2f}, C_{3f}. However, the feed tanks being remote from the reactors introduces input delays, while the transportation lags in the recycle streams cause the state delays. To meet the requirement in the 3rd stage, the product stream from Reactor 2 is slowly heated up to a certain temperature which is maintained in reactor 3. This is done through the τ time units of heated pipe between these two reactors. This introduces another transportation lag. (The distance between reactors 1 and 2 in time units is not significant.) Reaction is assumed to take place only inside the reactors.

A computer is available to implement the strict concentration control required *but only ONE composition measurement is available*; that of the product from reactor 2. The system can also operate under conventional PI control. Thus, the twofold problem

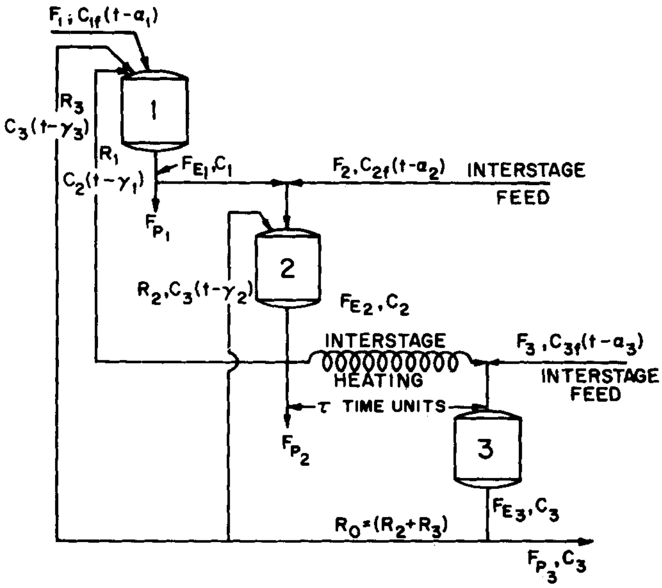


Figure 3. Network of 3 cascaded reactors.

is that of designing the controller to compensate for the multiple time delays as well as to maintain strict control of the concentrations when it has NO information about two out of these three required concentrations.

A material balance over the networks yields

$$v_1\frac{dC_1}{dt}=F_1C_{1f}(t-\alpha_1)+R_1C_2(t-\gamma_1)+R_3C_3(t-\gamma_3)-F_{E_1}C_1-v_1k_1C_1$$

$$v_2\frac{dC_2}{dt}=(F_{E_1}-F_{P_1})C_1+F_2C_{2f}(t-\alpha_2)+R_2C_3(t-\alpha_2)-F_{E_2}C_2-v_2k_2C_2$$

$$v_3\frac{dC_3}{dt}=(F_{E_2}-F_{P_2}-R_1)C_2(t-\tau)+F_3C_{3f}(t-\alpha_3)-F_{E_3}C_3-v_3k_3C_3$$

By defining the following variables

$$\theta_1=\frac{v_1}{F_{E_1}};\quad \theta_2=\frac{v_2}{F_{E_2}};\quad \theta_3=\frac{v_3}{F_{E_3}};$$

$$D_{a_1}=k_1\theta_1,\quad D_{a_2}=k_2\theta_2,\quad D_{a_3}=k_3\theta_3$$

$$\frac{R_1}{F_{E_1}}=\lambda_1;\quad \frac{R_3}{F_{E_1}}=\lambda_3;\quad \frac{F_1}{F_{E_1}}=(1-\lambda_1-\lambda_3)$$

since $F_{E_1}=F_1+R_1+R_3$

$$\frac{F_1-F_{P_1}}{F_{E_2}}=\mu_1;\quad \frac{R_2}{F_{E_2}}=\lambda_2;\quad \frac{F_2}{F_{E_2}}=(1-\mu_1-\lambda_2)$$

$$\frac{F_{E_2}-F_{P_2}-R_1}{F_{E_3}}=\mu_2\quad \frac{F_3}{F_{E_3}}=(1-\mu_2)$$

$$\left.\begin{aligned}\xi_i&=C_i-C_{is}\\ \nu_i&=C_{if}-C_{ifs}\end{aligned}\right\}i=1,2,3$$

where C_{is}, C_{ifs} denote the steady-state values of C_i and C_{if} we then obtain

$$\frac{d\xi_1(t)}{dt}=\frac{\lambda_1}{\theta_1}\xi_2(t-\gamma_1)+\frac{\lambda_3}{\theta_1}\xi_3(t-\gamma_3)-\frac{(1+D_{a1})}{\theta_1}\xi_1(t)+\frac{(1-\lambda_1-\lambda_3)}{\theta_1}\nu_1(t-\alpha_1)$$

$$\frac{d\xi_2(t)}{dt}=\frac{\mu_1}{\theta_2}\xi_1(t)+\frac{\lambda_2}{\theta_2}\xi_3(t-\gamma_2)-\frac{(1+D_{a2})}{\theta_2}\xi_2(t)+\frac{(1-\mu_1-\lambda_2)}{\theta_2}\nu_2(t-\alpha_2)$$

$$\frac{d\xi_3(t)}{dt}=\frac{\mu_2}{\theta_3}\xi_2(t-\tau)-\frac{(1+D_{a3})}{\theta_3}\xi_3(t)+\frac{(1-\mu_2)}{\theta_3}\nu_3(t-\alpha_3)$$

Since only the concentration in reactor 2 is measured ξ_2 is the only available information in the entire $\boldsymbol{\xi}$ vector. The vector can then be rearranged as below

$$\begin{matrix}\text{observed}\\ \text{subspace}\\ \text{unobserved}\\ \text{subspace}\end{matrix}\begin{bmatrix}\xi_2\\ \hline \xi_1\\ \xi_3\end{bmatrix}\Leftrightarrow\begin{bmatrix}x_1\\ \hline x_2\\ x_3\end{bmatrix}$$

accordingly, the input vector can be recast as

$$\begin{bmatrix}\nu_2\\ \nu_1\\ \nu_3\end{bmatrix}\Leftrightarrow\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

The state-space equations of the reactor network then becomes

$$\frac{dx_1}{dt}(t)=\frac{\mu_1}{\theta_2}x_2(t)+\frac{\lambda_2}{\theta_2}x_3(t-\gamma_2)-\frac{(1+D_{a_2})}{\theta_2}x_1(t)+\frac{(1-\mu_1-\lambda_2)}{\theta_2}u_1(t-\alpha_1)$$

$$\frac{dx_2}{dt}(t)=\frac{\lambda_1}{\theta_1}x_1(t-\gamma_1)+\frac{\lambda_3}{\theta_1}x_3(t-\lambda_3)-\frac{(1+D_{a_1})}{\theta_1}x_2(t)+\frac{(1-\lambda_1-\lambda_3)}{\theta_1}u_2(t-\alpha_2)$$

$$\frac{dx_3}{dt}=\frac{\mu_2}{\theta_3}x_1(t-\tau)-\frac{(1+D_{a_3})}{\theta_3}x_3(t)+\frac{(1-\mu_2)}{\theta_3}u_3(t-\alpha_3)\tag{27}$$

The restructuring done to the vector is for the purpose of having Eq. 27 in the form required for the observer design (cf. Definition 4 of Ogunnaike, 1981a) since this is clearly a case 1 problem.

For the purpose of simulation, let us choose:

$$\theta_1=1\qquad \theta_2=1/2\qquad \theta_3=1$$

$$D_{a_1}=1\qquad D_{a_2}=1/2\qquad D_{a_3}=3$$

$$\lambda_1=0.2\qquad \lambda_3=0.3$$

$$\mu_1=0.3\qquad \lambda_2=0.2\qquad \mu_2=0.4$$

and, for the delays, let

$$\gamma_1=1,\qquad \gamma_2=2,\qquad \gamma_3=4,\qquad \tau=3$$

$$\alpha_1=2\qquad \alpha_2=4\qquad \alpha_3=1$$

then, Eq. 27 immediately becomes

$$\frac{dx_1}{dt}(t)=-3x_1(t)+0.6x_2(t)+0.4x_3(t-2)+u_1(t-4)$$

$$\frac{dx_2}{dt}(t)=0.2x_1(t-1)-2x_2(t)+0.3x_3(t-4)+0.5u_2(t-2)$$

$$\frac{dx_3}{dt}(t)=0.4x_1(t-3)-0.4x_3(t)+0.6u_3(t-1)\tag{28}$$

If Eqs. 28 are written as

$$\dot{x}=\mathcal{A}x+\mathcal{B}u\qquad \text{then}$$

$$\mathcal{A}=\begin{bmatrix}-3 & 0.6 & 0.4B^2\\ 0.2B^1 & -2 & 0.3B^3\\ 0.4B^3 & 0 & -4\end{bmatrix}\quad \mathcal{B}=\begin{bmatrix}B^4 & 0 & 0\\ 0 & 0.5B^2 & 0\\ 0 & 0 & 0.6B^1\end{bmatrix}$$

Design of Time Delay Compensator

If Eqs. 28 are written as

$$\dot{x}=\sum_{i=0}^{4}A_i x(t-\rho_i)+\sum_{j=1}^{3}B_j u(t-\beta_j)\tag{29}$$

We observe that

$$A_0 = \begin{bmatrix} -3 & 0.6 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 0 & 0.4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.4 & 0 & 0 \end{bmatrix} \quad A_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.3 \\ 0 & 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.6 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Clearly, x^* is obtained from

$$\dot{x}^* = A^* x^* + B^* u \tag{30}$$

where

$$A^* = \begin{bmatrix} -3 & 0.6 & 0.4 \\ 0.2 & -2 & 0.3 \\ 0.4 & 0 & -4 \end{bmatrix} \text{ and } B^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.6 \end{bmatrix} \tag{31}$$

In designing the compensator, *we will first consider all the three composition measurements as available*,

$$\text{i.e., } C = I$$

and then consequently design an observer to supply estimates of the unavailable concentrations. Thus, were all composition measurements available, the compensator equations would be

$$\dot{x} = \sum_{i=1}^{4} A_i x(t - \rho_i) + \sum_{j=1}^{3} B_j u(t - \beta_j)$$

$$y = x$$

and

$$\dot{x}^* = A^* x^* + B^* u$$

$$y = x^*$$

as in Eqs. 28, 30 and 31.

Observer Design

We now turn to the problem of designing an observer for the system in Eq. 28 when the only available information y is given by

$$y = [1 \quad 0 \quad 0]x \qquad (\text{i.e., } y = x_1)$$

Following the observer design procedure outlined above,

$$\mathcal{A} = \left[\begin{array}{c|cc} -3 & 0.6 & 0.4B^2 \\ \hline 0.2B^1 & -2 & 0.3B^4 \\ 0.4B^3 & 0 & -4 \end{array}\right] \quad \mathcal{B} = \left[\begin{array}{ccc} B^4 & 0 & 0 \\ \hline 0 & 0.5B^2 & 0 \\ 0 & 0 & 0.6B^1 \end{array}\right]$$

$$\mathcal{A}_{uu} = \begin{bmatrix} -2 & 0.3B^4 \\ 0 & -4 \end{bmatrix}, \ \mathcal{A}_{uo} = \begin{bmatrix} 0.2B^1 \\ 0.4B^3 \end{bmatrix}, \ \mathcal{B}_u = \begin{bmatrix} 0 & 0.5B^2 & 0 \\ 0 & 0 & 0.6B^1 \end{bmatrix}$$

For observer eigenvalues -3 and -6 respectively estimating x_2 and x_3 the matrix K is

$$K = \begin{bmatrix} -3 & 0 \\ 0 & -6 \end{bmatrix}$$

Hence for a choice of $\rho = 2$, from Eq. 7 or

$$\phi = 2\mathcal{A}_{uu} - K$$

$$\phi = \begin{bmatrix} -4 & -0.6B^4 \\ 0 & -8 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 0.6B^4 \\ 0 & -2 \end{bmatrix}$$

For this particular case then the necessary matrix Ω is given by

$$\Omega = \left[\begin{array}{c|cc} -3 & 0.6 & 0.4B^2 \\ \hline 0.2B^1 & -1 & 0.6B^4 \\ 0.4B^2 & 0 & -2 \end{array}\right] \tag{32}$$

Thus, from Eq. 8 the observer equations can be written down direct as

$$\dot{\hat{z}}_1 = -3\hat{z}_1 + 0.2[y(t-1) - \eta(t-1)] + 0.5u_2(t-2)$$

$$\dot{\hat{z}}_2 = -6\hat{z}_2 + 0.4[y(t-3) - \eta(t-3)] + 0.6u_3(t-1)$$

with $\eta(t)$ produced from

$$\dot{v} = \Omega v$$

$$\eta = v_1$$

and Ω as in Eq. 32
The estimates are given by

$$\hat{x}_2 = 2\hat{z}_1$$

$$\hat{x}_3 = 2\hat{z}_2$$

The Compensator Simulator

Synthesizing $(y^* - y)$ may be carried out using

$$\dot{w} = \Lambda w + \mathcal{D}x + \mathcal{P}u$$

where in this particular case,

$$\Lambda = A^* = \begin{bmatrix} -3 & 0.6 & 0.4 \\ 0.2 & -2 & 0.3 \\ 0.4 & 0 & -4 \end{bmatrix}. \quad \text{Since } C = I$$

and

$$\mathcal{D} = \begin{bmatrix} 0 & 0 & 0.4(1 - B^2) \\ 0.2(1 - B^1) & 0 & 0.3(1 - B^4) \\ 0.4(1 - B^3) & 0 & 0 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} (1 - B^4) & 0 & 0 \\ 0 & 0.5(1 - B^2) & 0 \\ 0 & 0 & 0.6(1 - B^1) \end{bmatrix}$$

The full equations in their real-time implementation forms are

$$\dot{w}_1 = -3w_1 + 0.6w_2 + 0.4w_3 + 0.4\hat{x}_3(t) - 0.4\hat{x}_3(t-2) + u_1(t) - u_1(t-4)$$

$$\dot{w}_2 = 0.2w_1 - 2w_2 + 0.3w_3 + 0.2\hat{x}_1(t) - 0.2\hat{x}_1(t-1) + 0.3\hat{x}_3(t) - 0.3\hat{x}_3(t-4) + 0.5u_2(t) - 0.5u_2(t-2)$$

$$\dot{w}_3 = 0.4w_1 - 4w_3 + 0.4\hat{x}_1(t) - 0.4\hat{x}_1(t-3) + 0.6u_3(t) - 0.6u_3(t-1).$$

Performance Testing by Computer Simulation

To test the performance of our overall design scheme (compensator-plus-observer) let us consider the situation where at time $t = 0$ the required concentration set-points in the reactors are changed according to

$$x_{1_d} = 1.0$$

$$x_{2_d} = -0.75$$

$$x_{3_d} = 0.5$$

G_c consists of three single-loop PI controllers given by

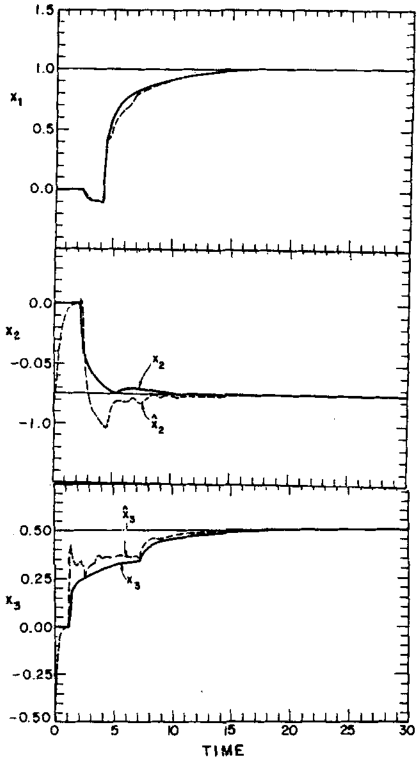


Figure 4. Concentration responses after set-point change with multidelay compensator —— when all concentration measurements are available; - - - - when observer estimates are used by compensated controller.

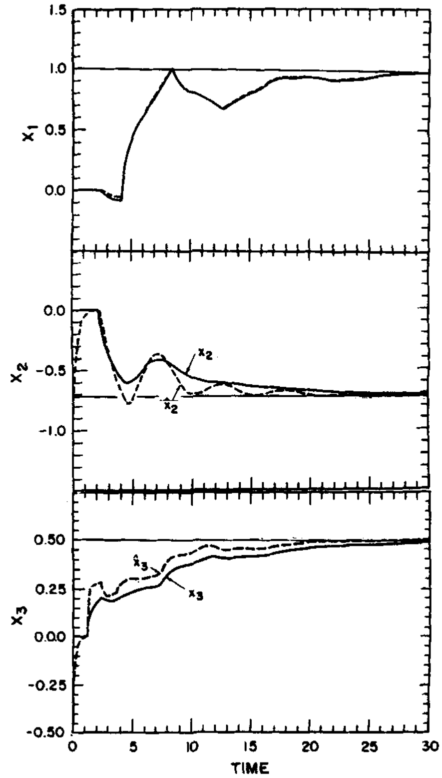


Figure 5. Concentration responses after set-point change under conventional PI control (no compensator) —— when all concentration measurements are available - - - - when observer estimates are used by compensated controller.

$$g_{c_i} = K_{c_i}\left(1 + \frac{1}{\tau_i s}\right)$$

The only information available to us, as we recall, is the concentration in the second reactor, i.e., x_1. Thus not knowing that the actual initial conditions $x_2(0)$, $x_3(0)$ are both zero, let our best guesses be

$$\hat{x}_2(0) = -1.0$$

$$\hat{x}_3(0) = -0.5$$

(Considering the setpoint changes, these initial estimates are in considerable error).

When the multidelay compensator is implemented for this example system according to Figure 2 the performance obtained is shown in Figures 4 for controller gain and reset values

$$\left.\begin{aligned} K_{c_i} &= 5.0 \\ \tau_i &= 2.0 \end{aligned}\right\} i = 1, 2, 3$$

The continuous lines in the figures represent the response of the system when *all* the three concentration measurements are made available to the controller. The dashed lines represent the response when the observer, given the above initial guesses, supplied the controller with estimates of the two unavailable concentration measurements.

We note that although starting from values in considerable error, the observer estimates quickly track the actual values. The observer performance in tracking x_3 is seen to be somewhat better than in tracking x_2. Recall that the observer eigenvalues are correspondingly −6 and −3, hence, the error $(x_3 - \hat{x}_3)$ is expected to die out faster than $(x_2 - \hat{x}_2)$.

On the whole, when one considers that the only measurement offered to the controller is x_1, the performance of the overall scheme, both in estimating the missing variables x_2 and x_3, and in quickly and efficiently driving all three concentrations to their desired setpoints is very impressive.

To illustrate how the dynamic performance degrades under conventional PI control (i.e., when only the observer is used and no compensation is made for the presence of time delays), Figure 5 shows the system response under these conditions for controller gain and reset values

$$K_{c_1} = 1.5, \qquad K_{c_i} = 2.0; \qquad i = 2, 3$$

$$\tau_1 = 4.0, \qquad \tau_i = 3.0; \qquad i = 2, 3$$

Note that the observer still performs reasonably well. However, the controller performance in driving the concentrations to their desired values is rather poor.

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NOTATION

A_i	= state vector matrix coefficients
A^*	= matrix coefficient in the undelayed version of a time delay system model
$\mathcal{A}$	= state vector operator matrix coefficient (time delay system model)
$\mathcal{A}_{oo}, \mathcal{A}_{ou}, \mathcal{A}_{uo}, \mathcal{A}_{uu}$	= partitions of $\mathcal{A}$
B	= backshift operator
B_j	= control vector matrix coefficient
B^*	= matrix coefficient in undelayed version of a time delay system model
$\mathcal{B}$	= control vector operator matrix coefficient (time delay system model)
$\mathcal{B}_o, \mathcal{B}_u$	= partitions of $\mathcal{B}$
C	= output matrix
C_1, C_2, C_{1f}, C_{2f}	= reacting fluid concentrations

$\mathcal{D}$	= operator matrix coefficient
D_{a_1}, D_{a_2}	= Damkohler numbers
$F_1, F_2, F_3, F_{E_1}, F_{E_2}, F_{E_3}, F_{P_1}, F_{P_2}$	= flow rates in reactor network
$G(s)$	= transfer function matrix
$G^*(s)$	= undelayed version of transfer function matrix $G(s)$
$G_c(s)$	= controller matrix
g_{c_i}	= elements of the controller matrix $G_c(s)$
I	= identity matrix
K_{c_i}	= controller gains
$\mathcal{P}$	= operator matrix coefficient
R	= matrix in observer formulation
s	= Laplace operator
u	= control vector
v	= state vector of complementary system
w	= state vector of compensator simulator
x_i, x, x^+	= state vectors of time delay systems
x^*	= undelayed system state vector
$\hat{x}$	= observer state estimates
y	= delay system output
y^*	= undelayed version of y
z	= observer variable

Greek Letters

α, β_i, γ	= time delays
ϕ	= matrix in the complementary system equation
η	= complementary system output
λ_i	= flow ratios
Λ	= matrix coefficient of the compensator simulator state variable
μ	= flow ratio
ρ_i	= time delay
ρ	= observer gain variable
$\sigma(\cdot)$	= spectrum of $(\cdot)$
ξ_i	= concentration deviation variable
θ_i	= reactor residence time
τ	= transportation lag in pipe
τ_i	= integral time
Ω	= matrix coefficient of the complementary system

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